

1.

(1):  $x^3$  could be 11 or 27 or 97... so not a unique value

(2):  $x^4$  could be 11 or 27 or 97... so not a unique value (also,  $x$  could be positive or negative)

Combining:  $x^3$  and  $x^4$  both are integers... so  $x$  cannot be irrational.  $x$  can be only 3.

Ans. (C)

2.

(1):  $x \neq 0$ , could be + or -ve.

(2)  $x = 0$  or -ve.

Comb...  $x$  is negative. Ans. (C)

3.

(1):  $5z = 2$ ,  $z = 2/5$ ;  $5z = 10$ ,  $z = 2$ ... NS

(2):  $3z = 2$ ,  $z = 2/3$ ;  $3z = 6$ ,  $z = 2$ ... NS

Combining: Subtract the two:  $2z = \text{even} - \text{even} = \text{even}$ ... so  $z$  has to be an integer... if  $z$  is an integer and  $5z$  is even,  $z$  has to be even. Ans. (C)

4.

We can first simplify the exponential expression in the question:

$$b^{a+1} - ba^b$$

$$b(b^a) - b(a^b)$$

$$b(b^a - a^b)$$

So we can rewrite this question then as is  $b(b^a - a^b)$  odd? Notice that if either  $b$  or  $b^a - a^b$  is even, the answer to this question will be no.

(1) SUFFICIENT: If we simplify this expression we get  $5a - 8$ , which we are told is odd. For the difference of two numbers to be odd, one must be odd and one must be even. Therefore  $5a$  must be odd, which means that  $a$  itself must be odd. To determine whether or not this is enough to dictate the even/oddness of the expression  $b(b^a - a^b)$ , we must consider two scenarios, one with an odd  $b$  and one with an even  $b$ :

$a$	$b$	$b(b^a - a^b)$	odd/even
3	1	$1(1^3 - 3^1) = -2$	even
3	2	$2(2^3 - 3^2) = -2$	even

It turns out that for both scenarios, the expression  $b(b^a - a^b)$  is even.

(2) SUFFICIENT: It is probably easiest to test numbers in this expression to determine whether it implies that  $b$  is odd or even.

$b$	$b^3 + 3b^2 + 5b + 7$	odd/even
2	$2^3 + 3(2^2) + 5(2) + 7 = 37$	odd
1	$1^3 + 3(1^2) + 5(1) + 7 = 16$	even

We can see from the two values that we plugged that only even values for  $b$  will produce odd values for the expression  $b^3 + 3b^2 + 5b + 7$ , therefore  $b$  must be even. Knowing that  $b$  is even tells us that the product in the question,  $b(b^a - a^b)$ , is even so we have a definitive answer to the question.

The correct answer is D, EACH statement ALONE is sufficient to answer the question.

5.

(1) INSUFFICIENT: We are told that  $5n/18$  is an integer. This does not allow us to determine whether  $n/18$  is an integer. We can come up with one example where  $5n/18$  is an integer and where  $n/18$  is **NOT** an integer. We can come up with another example where  $5n/18$  is an integer and where  $n/18$  **IS** an integer.

Let's first look at an example where  $5n/18$  is equal to the integer 1.

If  $\frac{5n}{18} = 1$ , then  $\frac{n}{18} = \frac{1}{5}$ . In this case  $n/18$  is NOT an integer.

Let's next look at an example where  $5n/18$  is equal to the integer 15.

If  $\frac{5n}{18} = 15$ , then  $\frac{n}{18} = 3$ . In this case  $n/18$  IS an integer.

Thus, Statement (1) is NOT sufficient.

(2) INSUFFICIENT: We can use the same reasoning for Statement (2) that we did for statement (1). If  $3n/18$  is equal to the integer 1, then  $n/18$  is NOT an integer. If  $3n/18$  is equal to the integer 9, then  $n/18$  IS an integer. **This tells us  $n$  is a multiple of 6.**

(1) AND (2) SUFFICIENT: If  $5n/18$  and  $3n/18$  are both integers, the difference of  $5n/18$  and  $3n/18$  will also be integer (integer – integer = integer)

So  $5n/18 - 3n/18 = 2n/18 = n/9 = \text{integer} \dots$   **$n$  is a multiple of 9...** So  $n$  is a multiple of both 6 and 9... so  $n$  is a multiple of 18.

Another way to understand this solution is to note that according to (1),  $n = (18/5) \cdot \text{integer}$ , and according to (2),  $n = 6 \cdot \text{integer}$ . In other words,  $n$  is a multiple of both  $18/5$  and  $6$ . The least common multiple of these two numbers is  $18$ . In order to see this, write  $6 = 30/5$ . The LCM of the numerators  $18$  and  $30$  is  $90$ . Therefore, the LCM of the fractions is  $90/5 = 18$ . **Again, the correct answer is C.**

6.

The possible values of  $n$  should be computed right away, to rephrase and simplify the question. Note that  $n$  consecutive positive integers that sum to  $45$  have a mean of  $45/n$ , which is also the median of the set; therefore, the set must be arranged around  $45/n$ . Also, any set of consecutive integers must have either an integer mean (if the number of integers is odd) or a mean that is an integer  $+ 1/2$  (if the number of integers is even). So, if we compute  $45/n$  and see that it is neither an integer nor an integer  $+ 1/2$ , then we can eliminate this possibility right away. Setting up a table that tracks not only the value of  $n$  but also the value of  $45/n$  is useful.

$n$	$45/n$	$n$ positive consecutive integers summing to $45$
1	45	45
2	22.5	22, 23
3	15	14, 15, 16
4	11.25	none
5	9	7, 8, 9, 10, 11
6	7.5	5, 6, 7, 8, 9, 10
7	$6 \frac{3}{7}$	none
8	$5 \frac{5}{8}$	none
9	5	1, 2, 3, 4, 5, 6, 7, 8, 9
10	4.5	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 -- but this doesn't work, because not all are positive integers
...	...	impossible (the set will include negative integers, if an integer set can be found at all)

(1) INSUFFICIENT: If  $n$  is even,  $n$  could be either  $2$  or  $6$ . Statement (1) is NOT sufficient.

Alternatively, to find these values algebraically, you can use the following procedure.

The sum of two consecutive integers can be represented as  $n + (n + 1) = 2n + 1$

The sum of three consecutive integers  $= n + (n + 1) + (n + 2) = 3n + 3$

The sum of four consecutive integers  $= 4n + 6$

The sum of five consecutive integers  $= 5n + 10$

The sum of six consecutive integers  $= 6n + 15$

Since the expressions  $2n + 1$  and  $6n + 15$  can both yield  $45$  for integer values of  $n$ ,  $45$  can be the sum of two or six consecutive integers.

(2) INSUFFICIENT: If  $n < 9$ ,  $n$  could again take on either of the values  $2$  or  $6$  (or  $3$  or  $5$  according to the table or the expressions above)

(1) and (2) INSUFFICIENT: if we combine the two statements,  $n$  must be even and less than 9, so  $n$  could still be either of the values: 2 or 6.

The correct answer is E.

7.

(1) SUFFICIENT: Statement(1) tells us that  $x > 2^{34}$ , so we want to prove that  $2^{34} > 10^{10}$ . We'll prove this by manipulating the expression  $2^{34}$ .

$$2^{34} = (2^4)(2^{30})$$

$$2^{34} = 16(2^{10})^3$$

Now  $2^{10} = 1024$ , and 1024 is greater than  $10^3$ . Therefore:

$$2^{34} > 16(10^3)^3$$

$$2^{34} > 16(10^9)$$

$$2^{34} > 1.6(10^{10}).$$

Since  $2^{34} > 1.6(10^{10})$  and  $1.6(10^{10}) > 10^{10}$ , then  $2^{34} > 10^{10}$ .

(2) SUFFICIENT: Statement (2) tells us that that  $x = 2^{35}$ , so we need to determine if  $2^{35} > 10^{10}$ . Statement (1) showed that  $2^{34} > 10^{10}$ , therefore  $2^{35} > 10^{10}$ .

The correct answer is D.

8.

$$X - 2Y < -6 \Rightarrow -X + 2Y > 6$$

Combined  $X - Y > -2$ , we know  $Y > 4$

$$X - Y > -2 \Rightarrow -2X + 2Y < 4$$

Combined  $X - 2Y < -6$ , we know  $-X < -2 \Rightarrow X > 2$

Therefore,  $XY > 0$

Answer is C

9.

This question cannot be rephrased since it is already in a simple form.

(1) INSUFFICIENT: Since  $x^2$  is positive whether  $x$  is negative or positive, we can only determine that  $x$  is not equal to zero;  $x$  could be either positive or negative.

(2) INSUFFICIENT: By telling us that the expression  $x \cdot |y|$  is not a positive number, we know that it must either be negative or zero. If the expression is negative,  $x$  must be negative ( $|y|$  is never negative). However if the expression is zero,  $x$  or  $y$  could be zero.

(1) AND (2) INSUFFICIENT: We know from statement 1 that  $x$  cannot be zero, however, there are still two possibilities for  $x$ :  $x$  could be positive ( $y$  is zero), or  $x$  could be negative ( $y$  is anything).

The correct answer is E.

10.

(1) INSUFFICIENT: This expression provides only a range of possible values for  $x$ .

(2) SUFFICIENT: Absolute value problems often -- **but not always** -- have multiple solutions because the expression *within* the absolute value bars can be either positive or negative even though the absolute value of the expression is always positive. For example, if we consider the equation  $|2 + x| = 3$ , we have to consider the possibility that  $2 + x$  is already positive and the possibility that  $2 + x$  is negative. If  $2 + x$  is positive, then the equation is the same as  $2 + x = 3$  and  $x = 1$ . But if  $2 + x$  is negative, then it must equal  $-3$  (since  $|-3| = 3$ ) and so  $2 + x = -3$  and  $x = -5$ .

So in the present case, in order to determine the possible solutions for  $x$ , it is necessary to solve for  $x$  under both possible conditions.

For the case where  $x > 0$ :

$$\begin{aligned}x &= 3x - 2 \\ -2x &= -2 \\ x &= 1\end{aligned}$$

For the case when  $x < 0$ :

$$\begin{aligned}x &= -1(3x - 2) \text{ We multiply by } -1 \text{ to make } x \text{ equal a negative quantity.} \\ x &= 2 - 3x \\ 4x &= 2 \\ x &= 1/2\end{aligned}$$

Note however, that the second solution  $x = 1/2$  contradicts the stipulation that  $x < 0$ , hence there is no solution for  $x$  where  $x < 0$ . Therefore,  $x = 1$  is the only valid solution for

(2).

The correct answer is B.

11.

The question "Is  $|x|$  less than 1?" can be rephrased in the following way.

Case 1: If  $x > 0$ , then  $|x| = x$ . For instance,  $|5| = 5$ . So, if  $x > 0$ , then the question becomes "Is  $x$  less than 1?"

Case 2: If  $x < 0$ , then  $|x| = -x$ . For instance,  $|-5| = -(-5) = 5$ . So, if  $x < 0$ , then the question becomes "Is  $-x$  less than 1?" This can be written as follows:

$-x < 1$ ? or, by multiplying both sides by  $-1$ , we get  $x > -1$ ?

Putting these two cases together, we get the fully rephrased question:  
"Is  $-1 < x < 1$  (and  $x$  not equal to 0)?"

Another way to achieve this rephrasing is to interpret absolute value as distance from zero on the number line. Asking "Is  $|x|$  less than 1?" can then be reinterpreted as "Is  $x$  less than 1 unit away from zero on the number line?" or "Is  $-1 < x < 1$ ?" (The fact that  $x$  does not equal zero is given in the question stem.)

(1) INSUFFICIENT: If  $x > 0$ , this statement tells us that  $x > x/x$  or  $x > 1$ . If  $x < 0$ , this statement tells us that  $x > x/-x$  or  $x > -1$ . This is not enough to tell us if  $-1 < x < 1$ .

(2) INSUFFICIENT: When  $x > 0$ ,  $x > x$  which is not true (so  $x < 0$ ). When  $x < 0$ ,  $-x > x$  or  $x < 0$ . Statement (2) simply tells us that  $x$  is negative. This is not enough to tell us if  $-1 < x < 1$ .

(1) AND (2) SUFFICIENT: If we know  $x < 0$  (statement 2), we know that  $x > -1$  (statement 1). This means that  $-1 < x < 0$ . This means that  $x$  is definitely between  $-1$  and  $1$ .

The correct answer is C.

12.

(1) SUFFICIENT: We can combine the given inequality  $r + s > 2t$  with the first statement by adding the two inequalities:

$$r + s > 2t$$

$$\underline{t > s}$$

$$r + s + t > 2t + s$$

$$r > t$$

(2) SUFFICIENT: We can combine the given inequality  $r + s > 2t$  with the second statement by adding the two inequalities:

$$r + s > 2t$$

$$\underline{r > s}$$

$$2r + s > 2t + s$$

$$2r > 2t$$

$$r > t$$

The correct answer is D.

13. (1) INSUFFICIENT: Since this equation contains two variables, we cannot determine the value of  $y$ . We can, however, note that the absolute value expression  $|x^2 - 4|$  must be greater than or equal to 0. Therefore,  $3|x^2 - 4|$  must be greater than or equal to 0, which in turn means that  $y - 2$  must be greater than or equal to 0. If  $y - 2 \geq 0$ , then  $y \geq 2$ .

(2) INSUFFICIENT: To solve this equation for  $y$ , we must consider both the positive and negative values of the absolute value expression:

$$\text{If } 3 - y > 0, \text{ then } 3 - y = 11$$

$$y = -8$$

If  $3 - y < 0$ , then  $3 - y = -11$

$$y = 14$$

Since there are two possible values for  $y$ , this statement is insufficient.

(1) AND (2) SUFFICIENT: Statement (1) tells us that  $y$  is greater than or equal to 2, and statement (2) tells us that  $y = -8$  or 14. Of the two possible values, only 14 is greater than or equal to 2. Therefore, the two statements together tell us that  $y$  must equal 14.

The correct answer is C.

**14.**

If  $x$  is +ve,  $y$  is +ve

If  $x$  is -ve,  $y$  is 0

If  $x$  is 0,  $y$  is 0.

**So  $y$  is not negative**

For 1,  $x < 0$ ,  $x + |x| = 0$

For 2,  $y < 1$ , noticed that  $y$  is an integer,  $y$  only can be 0.

Answer is D

**15.**

**SQUARE is never negative... square  $\geq 0$  always.**

Given equation can be written as, Is  $x^2 - 2x + 3y > 0$ ?

i.  $2x - 3y = -2$

The given equation becomes  $x^2 - (2x - 3y) = x^2 + 2$ , which is always  $> 0$   
(SUFF)

st1:

$$2x - 3y = -2$$



since  $x^2$  will always be +ve,  $(2x - 3y)$  will always be less than  $x^2$ . SUFFICIENT

statement 1:  $2x - 3y = -2$

since for all values of  $x$ ,  $x^2 \geq 0$

hence  $2x - 3y < x^2$

ii.  $x > 2$  and  $y > 0$

Given equation can be written as  $x(x - 2) + 3y$ , which is also always  $> 0$ .

$x > 2$  and  $y > 0$

when  $x > 2$ , for any value of  $x$ ,  $x^2$  will always be greater than  $2x$

$\Rightarrow x^2 > 2x$

and since  $y$  is +ve

$\Rightarrow x^2 > 2x - 3y$ .

$x > 2$  and  $y > 0$

since  $2x < x^2$  for  $x > 2$

so  $2x - 3y < 2x$  for  $y > 0$

$\Rightarrow 2x - 3y < x^2$

sufficient

answer D.

**16:**

Each statement alone is not sufficient.

Combining... add the two statements...

(1) + (2), we can know that  $z > 0$ , then,  $m > 3z > 0$ . Together,  $m + z > 0$  Answer is C

**17.**

When solve such kind of questions, we just need to know the ratio one price to another price.  
It is time waste to calculate one by one.

Both two statements do not give the information, as well as their combination.

Answer is E

18.

"one kilogram of a certain coffee blend consists of X kilogram of type I and Y kilogram of type II" means that  $X+Y=1$

Combined  $C=6.5X+8.5Y$ , we get:

$$X=(8.5-C)/2, Y=(C-6.5)/2$$

$$\text{Combined } C \geq 7.3, X=(8.5-C)/2 < 1.2/2=0.6$$

Answer is B

19.

It is somewhat tricky.

Usually, we need two equations to solve two variables.

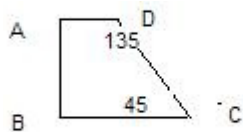
For example, in this question, from 1,  $x=y=6$ , from 2,  $21x+23y=130$ , the answer should be C.

Actually, the variables in such questions should be integers. Thus, hopefully, we can solve them with only one equation.

$21x+23y=130$ , we try  $x=1, 2, 3, 4, 5$ ..and find that only  $x=4, y=2$  can fulfill the requirements.

Answer is B.

20.



The figure can fulfill the entire requirement, but there is no any angle that equal to 60.

$$\text{Sum of 4 angles} = (n - 2) * 180 = 360$$

$$\text{From 1: sum of the remaining angles are } 360 - 2*90 = 180$$

$$\text{From 2: either } x + 2x = 180 \Rightarrow x = 60$$

Or  $x = 90/2 = 45$  and  $y = 180 - 45 = 135$ .

Answer is E

**21.**

The perimeter of a triangle is equal to the sum of the three sides.

(1) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

(2) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

Together, the two statements are SUFFICIENT. Triangle ABC is an isosceles triangle which means that there are theoretically 2 possible scenarios for the lengths of the three sides of the triangle: (1)  $AB = 9$ ,  $BC = 4$  and the third side,  $AC = 9$  **OR** (2)  $AB = 9$ ,  $BC = 4$  and the third side  $AC = 4$ .

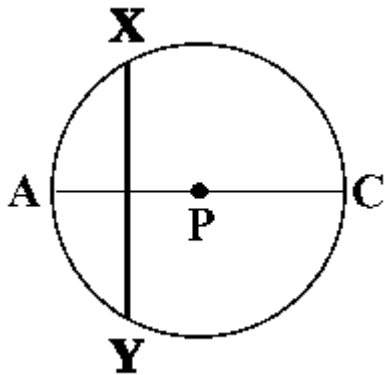
These two scenarios lead to two different perimeters for triangle ABC, HOWEVER, upon careful observation we see that the second scenario is an IMPOSSIBILITY. A triangle with three sides of 4, 4, and 9 is not a triangle. Recall that any two sides of a triangle must sum up to be greater than the third side.  $4 + 4 < 9$  so these are not valid lengths for the side of a triangle.

Therefore the actual sides of the triangle must be  $AB = 9$ ,  $BC = 4$ , and  $AC = 9$ . The perimeter is 22.

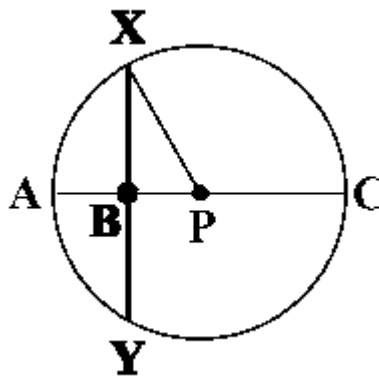
The correct answer is C.

22.

Since no picture is given in the problem, draw it. Below, find the given circle with center  $P$  and cord  $XY$  bisecting radius  $AP$ .



Although, the picture above is helpful, drawing in an additional radius is often an important step towards seeing the solution. Thus, we will add to the picture by drawing in radius  $XP$  as shown below.



Since  $XY$  bisects radius  $AP$  at point  $B$ , segment  $BP$  is half the length of any radius.

Since  $BP$  is half the length of radius  $XP$ , right triangle  $XPB$  must be a 30-60-90 triangle with sides in the ratio of  $1 : \sqrt{3} : 2$ .

Therefore, finding the length of any side of the triangle, will give us the lengths of the other two sides.

Finding the length of radius  $XP$  will give us the length of  $XB$ , which is half the length of cord

XY. Thus, in order to answer the question--**what is the length of cord XY?**--we need to know only one piece of information:

The length of the radius of the circle.

Statement (1) alone tells us that the circumference of the circle is twice the area of the circle. Using this information, we can set up an equation and solve for the radius as follows:

$$\text{Circum} = 2 \times \text{Area}$$

$$2\pi r = 2(\pi r^2)$$

$$2r = 2r^2$$

$$r = r^2$$

$$r = 1$$

Therefore Statement (1) alone is sufficient to answer the question.

Statement (2) alone tells us that the length of Arc  $XAY = \frac{2\pi}{3}$ .

Arc  $XAY$  is made up of arc  $XA$  + arc  $AY$ .

Given that triangle  $XPB$  is a 30:60: 90 triangle, we know that  $\angle XPA = 60$  degrees and can deduce that  $\angle APY = 60$  degrees as well. Therefore Arc  $XAY = 120$  degrees or  $1/3$  of the circumference of the circle. Using this information, we can solve for the radius of the circle by setting up an equation as follows:

$$\text{Arc } XAY = \frac{1}{3} \text{ Circum}$$

$$\frac{2\pi}{3} = \frac{1}{3}(2\pi r)$$

$$2\pi = 2\pi r$$

$$1 = r$$

Therefore, Statement (2) alone is also sufficient to answer the question. The correct answer is D, each statement ALONE is sufficient.

23.

Firstly, we assume that  $a*b > 0$ . Let  $a=1$ ,  $b=2$ , then  $(-a,b)=(-1,2)$ ,  $(-b,a)=(-2,1)$ , the two points are in the second quadrant. From 2),  $ax > 0$ ,  $x$  and  $a$  are both positive or both negative, as well as the  $-x$  and  $-a$ . From 1),  $xy > 0$ ,  $x$  and  $y$  are both positive or both negative, while  $-x$  and  $y$  are different. Above all, point  $(-a,b)$  and  $(-x,y)$  are in the same quadrant. Then we assume that  $a*b < 0$ . Let  $a=1$ ,  $b=2$ , then  $(-a,b)=(-1,2)$ ,  $(-b,a)=(-2,-1)$ , in different quadrants. This is conflict to the question, need no discussion. Answer is C

24.

To find the area of equilateral triangle  $ABC$ , we need to find the length of one side. The area of an equilateral triangle can be found with just one side since there is a known ratio between the side and the height (using the 30: 60: 90 relationship). Alternatively, we can find the area of an equilateral triangle just knowing the length of its height.

(1) INSUFFICIENT: This does not give us the length of a side or the height of the equilateral triangle since we don't have the coordinates of point A.

(2) SUFFICIENT: Since  $C$  has an  $x$ -coordinate of 6, the height of the equilateral triangle must be 6.

The correct answer is B.

25.

This is an AP... common difference either positive or negative. There are 15 terms, so the 8<sup>th</sup> term will be the median. 7 terms will be less than the median and 7 terms will be more than the median. If median is 10, then we know that 7 terms are more than 10 and 7 terms are less than 10. Ans. B